Rough Sea Surface Reflection Coefficient Estimation and Its Implication On Hydrophone-Only Pre-Stack Deghosting

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Summary

Deghosting a hydrophone-only measurement requires accurate description of seismic wavefield scattering at the sea surface. In recent years, attempts have been made to deghost hydrophone-only data using a frequency and angle dependent sea surface reflection coefficient. Nevertheless, rough sea surface reflection coefficients have both specular and non-specular contributions. In pre-stack hydrophone-only deghosting, the non-specular contributions are often assumed to be negligible. In this paper, we characterize the sea surface scattering using numerical modelling and quantify the contributions of both specular and non-specular reflections from two different realistic sea states. The results show that the magnitude of the specular reflection coefficient decreases with frequency and increases with angle. Moreover, the rougher the sea surface, the higher the magnitude of the non-specular contributions.

Introduction

Proper pre-stack deghosting on the receiver-side is possible when both pressure and vertical particle velocity (or acceleration) measurements are available (Fokkema and van den Berg, 1993). However, when only the pressure measurement is available, then additional information is required in order to perform accurate receiver-side deghosting (Asgedom et al., 2016). This additional information which describes what happens to the up-going wavefield between the streamer and the sea surface is often expressed in the form of a Ghost function. The crucial part of the Ghost function is the sea surface reflectivity that describes the scattering behavior of the sea surface.

The sea surface reflectivity (or the plane wave reflection coefficient) provides a physical model to describe seismic scattering at the sea surface. Numerically, the sea surface reflectivity can be computed for a given rough sea surface using the Helmholtz Kirchhoff Integral (HKI) or Kirchhoff Approximation (KA) (Thorsos, 1987; Orji et al., 2011). In this paper the two approaches are compared quantitatively in terms of the sea surface plane wave reflection coefficient matrix from very rough and slightly rough sea surfaces. The validity of the KA is re-examined in terms of the specular and non-specular reflections from the rough sea surfaces. The statistical behavior of the sea surface scattering is analyzed by computing the total and coherent sea surface plane wave reflection coefficients.

Theory

The sea surface reflectivity is the scattered pressure wavefield recorded by single sensors (i.e., without group forming) as a result of ideal sources (i.e. Dirac in space and time). Utilizing HKI, the sea surface reflectivity $R_{sea}(\omega, \boldsymbol{r}_r|\boldsymbol{r}_s)$ for a given angular frequency ω, sources at $\boldsymbol{r}_s = (\boldsymbol{x}_s, z_s)$ and receivers at $\boldsymbol{r}_r = (\boldsymbol{x}_r, z_r)$ can be given by (Thorsos, 1987; Orji et al., 2011)

$$R_{sea}(\omega, \boldsymbol{r}_r|\, \boldsymbol{r}_s) = -\int_{S_{fs}} \left[G(\omega, \boldsymbol{r}_r|\boldsymbol{r}_{fs})[\boldsymbol{\nabla}P(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_s).\boldsymbol{n}_{fs}]\right] dS_{fs}, \quad (1)$$

where, $G(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_r)$ is the free-space Green functions with sources at $\boldsymbol{r}_{fs} = (\boldsymbol{x}_{fs}, z_{fz})$ on the sea surface and receivers at $\boldsymbol{r}_r$. Moreover, $\boldsymbol{\nabla}P(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_s)$ and $\boldsymbol{n}_{fs}$ are the pressure gradient and the normal vector at the sea surface S_{fs}, respectively. To compute R_{sea} using Eq. (1), we first need to know the pressure gradient $\boldsymbol{\nabla}P(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_s)$ at the sea surface. Using the fact that the pressure wavefield vanishes at the sea surface, the pressure gradient at the sea surface can be obtained by solving an integral inversion problem cast in the form of a Fredholm integral of either the first or second kind (e.g., Thorsos, 1987). In this paper, a Fredholm integral of the first kind is used.

Exact calculation of the sea surface reflectivity using the HKI is computationally intensive. Thus, several approximate methods that are computationally less demanding have been proposed (Thorsos, 1987). The Kirchhoff Approximation (KA), which assumes that the sea surface is locally planar and hence approximates the pressure gradient at the sea surface as twice the incident wavefield (i.e. $\boldsymbol{\nabla}P(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_s) \approx 2\boldsymbol{\nabla}P_{inc}(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_s)$), is one such approximate method. Consequently, the sea surface reflectivity as a result of KA may be written as

$$R_{sea}^{KA}(\omega, \boldsymbol{r}_r|\, \boldsymbol{r}_s) = -2\int_{S_{fs}} \left[G(\omega, \boldsymbol{r}_r|\boldsymbol{r}_{fs})[\boldsymbol{\nabla}P_{inc}(\omega, \boldsymbol{r}_{fs}|\boldsymbol{r}_s).\boldsymbol{n}_{fs}]\right] dS_{fs}\,. \quad (2)$$

Using acoustic reciprocity of the time convolution (Fokkema and van den Berg, 1993), the sea surface reflectivity and the incident pressure wavefield can be related using the plane wave reflection coefficient matrix $\hat{R}_{coef}$ as follows

$$\hat{R}_{sea}(\omega, \boldsymbol{k}_r, z_r \mid \boldsymbol{k}_s, z_s) = \int_{-\infty}^{\infty} \hat{R}_{coef}(\omega, -\boldsymbol{k}_{sep}, z_{sep} \mid \boldsymbol{k}_r,\, z_r)\hat{P}_{inc}(\omega, \boldsymbol{k}_{sep}, z_{sep} \mid \boldsymbol{k}_s, z_s) d\boldsymbol{k}_{sep}\,, \quad (3)$$

where, $\boldsymbol{k}_r$, $\boldsymbol{k}_s$ and $\boldsymbol{k}_{sep}$ are the lateral wavenumber vectors at the receiver, source and separation level (i.e., hypothetical surface where the reflection coefficient is computed), respectively. Equation 3 implies that the plane wave reflection coefficient matrix is the result of removing (or deconvolving) the propagation effects and the ideal point nature of the source from the sea surface reflectivity.

The plane wave reflection coefficient matrix at the mean sea level (i.e., $z_r = z_{sep} = 0$) can be computed by employing the Weyl plane wave expansion of the free-space Green functions, $G(\omega, \boldsymbol{r}_r | \boldsymbol{r}_{fs}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\boldsymbol{k}_r \frac{\exp\{i\boldsymbol{k}_r.(\boldsymbol{x}_r - \boldsymbol{x}_{fs}) + ik_z^r|z_r - z_{fs}|\}}{-2ik_z^r}$ with k_z^r being the vertical wavenumber for the receiver coordinate. Substituting Eq. 3 into the wavenumber-frequency transforms of Eqs. 1 and 2, we obtain

$$\hat{R}_{coef}(\omega, \boldsymbol{k}_r | \boldsymbol{k}_s) = -\frac{1}{(2\pi)^2}\int_{S_{fs}} \left[\left[\frac{\exp\{-i(k_z^r z_{fs} + \boldsymbol{k}_r.\boldsymbol{x}_{fs})\}}{k_z^r}\right] k_z^s \exp\{ik_z^s z_s\} \boldsymbol{\nabla} P(\omega, \boldsymbol{r}_{fs} | \boldsymbol{k}_s).\boldsymbol{n}_{fs}\right] dS_{fs}, \quad \text{(4a)}$$

$$\hat{R}_{coef}^{KA}(\omega, \boldsymbol{k}_r | \boldsymbol{k}_s) = -\frac{1}{(2\pi)^2 k_z^r}\int_{S_{fs}} \left[\exp\{-i[k_z^r + k_z^s]z_{fs} + i[\boldsymbol{k}_s - \boldsymbol{k}_r].\boldsymbol{x}_{fs}\} \left[\frac{dz_{fs}}{d\boldsymbol{x}_{fs}}\boldsymbol{k}_s + k_z^s\right]\right] d\boldsymbol{x}_{fs}\,. \quad \text{(4b)}$$

For a finite length sea surface, the computation of the plane wave reflection coefficient matrix based on Eqs. 4a or 4b must be normalized by $\frac{A_{fs}}{(2\pi)^2}$, where A_{fs} is the surface area of the sea surface. This normalization keeps the magnitude of the plane wave reflection coefficient between 0 and 1.

When the exact sea surface height variation is not known but its statistical parameters are available, the plane wave reflection coefficient matrix can be estimated in a statistical manner. For a finite size sea surface with a height variation following a Gaussian distribution with zero mean and standard deviation of σ, the coherent plane wave reflection coefficient matrix at the mean sea level based on the KA is given by (Thorsos, 1987)

$$\hat{R}_{coh}^{KA}(\omega, \boldsymbol{k}_r | \boldsymbol{k}_s) = \langle \hat{R}_{coef}^{KA}(\omega, \boldsymbol{k}_r | \boldsymbol{k}_s) \rangle = \exp\{-2[k_z^s\sigma]^2\} \hat{R}_{coef}^{Flat}(\omega, \boldsymbol{k}_r | \boldsymbol{k}_s)\,, \quad \text{(5)}$$

where, $\langle \quad \rangle$ is an expectation operator and $\hat{R}_{coef}^{Flat}$ is a finite length flat sea surface plane wave reflection coefficient matrix.

Numerical Examples

We consider a 2D scattering problem from two sea states with significant wave heights of 4.8m (i.e., very rough) and 2.1m (i.e., slightly rough). The very rough case represents a sea state that is considered close to the limit for safe towed streamer acquisition. The sea surfaces are 1D and they are generated from the Pierson-Moskowitz spectra with dominant wavelengths of 205.2m and 91.2m, respectively. To compute the sea surface reflectivities, we used 500 receivers separated by 3m and placed at a depth of 20m and a single source at a depth of 1km. A shot gather containing the sea surface reflectivity, computed using the HKI, is shown in Figs. 1a and 1b. Observe that the rougher the sea surface, the larger the undulation of the reflection events (i.e., the main energy peak) and the stronger the diffraction events (i.e., the weak amplitude hyperbolic features).

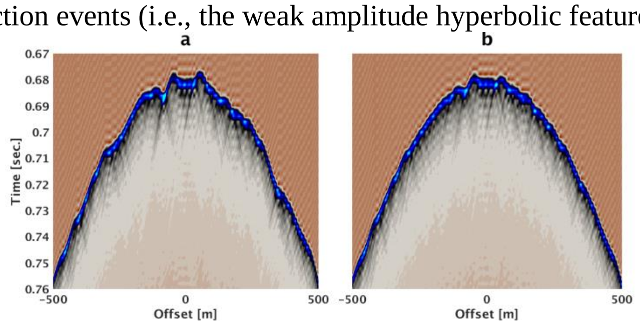

***Figure 1** Sea surface reflectivity shot gather for the very rough sea surface (a) and slightly rough sea surface (b).*

The sea surface reflectivity, unlike the plane wave reflection coefficient matrix, includes the effects of the source and receiver depths as well as the monopole nature of the source (cf. Eq. 3). To characterize the scattering at the sea surface, however, it is sufficient to analyse the plane wave reflection coefficient matrix. The total plane wave reflection coefficient can be decomposed into specular (i.e., mirror like reflections) and non-specular (i.e., the diffractions) parts. In order for energy conservation to be satisfied, the specular and non-specular contributions of the sea surface reflection coefficient must be complimentary to each other (i.e., the sum of specular and non-specular reflection coefficients for a given frequency are equal to one). Figures 2a – 2d, show the magnitude of the plane wave reflection coefficient matrix for vertical incidence (i.e., angle of incidence is zero) computed based on HKI and KA for the very rough and slightly rough sea surfaces. In Figs. 2a – 2d, the dominant energy at the vertical scattering angle represents the specular contribution. The non-specular contributions are in the directions away from the vertical scattering (i.e., directions different from 0^o). At lower frequencies, the plane wave reflection coefficient matrix is dominated by specular contributions. However, as the frequency increases the contribution of the non-specular reflections increase whilst the specular contributions decrease. Furthermore, the rougher the sea surfaces the higher the non-specular reflection contributions. Here, it is pertinent to note that, the plane wave reflection coefficient matrices from HKI and KA are similar around the specular reflection but they show differences for the non-specular regions. This is because; KA does not take into account multiple scattering and shadowing effects (Thorsos, 1987).

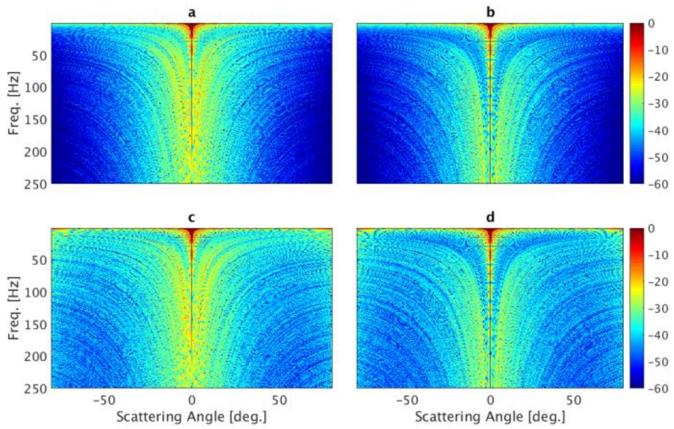

Figure 2 *Plane wave reflection coefficient matrices computed using HKI for the very rough (a) and slightly rough (b) sea surfaces. The plane wave reflection coefficient matrices based on KA for the very rough and slightly rough sea surfaces are shown in (c) and (d), respectively.*

To quantify the accuracy of the KA in predicting the specular plane wave reflection coefficient, we extracted the specular reflection coefficient (i.e., for vertical incidence, vertical scattering, and all frequencies) based on HKI and KA (cf. Fig. 3a). Observe that the results from the very and slightly rough sea surfaces converge at low frequencies and tends to 1. This validates flat sea surface assumption at these low frequencies. The specular reflection coefficient at 100Hz extracted for different scattering angles (with the condition that scattering angles = incidence angles) is shown in Fig. 3b. Figures 3a and 3b confirm that the specular reflection coefficient magnitude reduces with increasing frequency and increases with increasing angle (cf. Orji et al., 2013).

Statistically, the total plane wave reflection coefficient can also be decomposed into coherent and incoherent contributions. The coherent reflection coefficient provides a statistically robust prediction of the specular reflection coefficient. In order to check the accuracy of the KA in predicting both coherent and incoherent contributions of the sea surface scattering, we computed the total reflection coefficient (i.e., both coherent and incoherent) using a Monte Carlo simulation of 1000 iterations (i.e., sea surface realizations). Furthermore, we computed the coherent contribution of KA analytically using Eq. 6. Figures 4a and 4b show the coherent plane wave reflection coefficient based on KA and the total plane wave reflection coefficient based on HKI and KA (i.e., using Monte Carlo simulation). Fig. 4a and 4b are computed respectively for the very and slightly rough sea surfaces at 100 Hz and vertical incidence. From Figs. 4a and 4b, we observe that the incoherent reflection coefficients (i.e., the difference between the total and the coherent reflection coefficients) of the two sea surfaces considered are not negligible in magnitude. Moreover, KA accurately predicts the sea surface reflection coefficient around the coherent reflections, but fails to predict correctly the incoherent reflections that are away from the coherent reflection region.

Conclusion

Pre-stack hydrophone-only deghosting requires accurate knowledge of the sea surface reflectivity (or the plane wave reflection coefficient matrix). This demands accurate prediction of both specular and non-specular scattering from the sea surface. The specular plane wave reflection coefficient reduces with frequency and increases with angle. Moreover, the rougher the sea surfaces the higher the reduction of the specular reflection coefficient with frequency, which implies an increasing non-specular contribution. The KA makes a good approximation of the sea surface specular plane wave reflection coefficient but fails to correctly predict the non-specular reflections away from the specular vicinity. Results from very rough and slightly rough sea surfaces converge at low frequencies and tend towards a magnitude of 1, validating the flat sea surface assumption at these frequencies. Finally, we observe that the specular reflection coefficient can be approximated analytically using the KA provided the sea surface height standard deviation is known and the sea surface height distribution satisfies a Gaussian distribution.

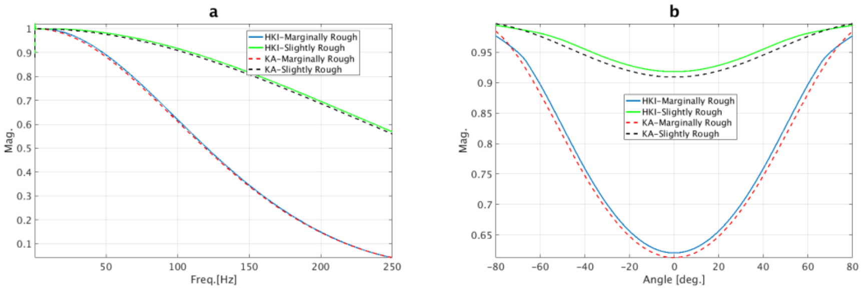

Figure 3 *Specular plane wave reflection coefficient as function of frequency at vertical incidence (a) and angle at 100 Hz (b).*

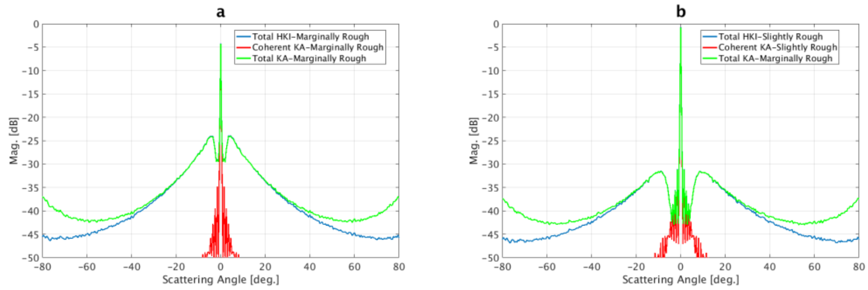

Figure 4 *Total reflection coefficient based on HKI, KA, and coherent KA reflection coefficient for the very rough (a) and slightly rough (b) sea surfaces.*

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