An effective multi-parameter Full Waveform Inversion in acoustic anisotropic media

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Summary

Multi-parameter Full Waveform Inversion (FWI) suffers from leakage among different medium properties. Here we discuss a practical multi-parameter FWI solution that minimizes leakage and retrieves the long-wavelength features of the anisotropic earth model. Our algorithm is based on a regularization of the objective function and a specific parameterization of an acoustic Transversely Isotropic (TI) medium consisting of the vertical velocity, and Thomsen parameters epsilon and delta. We show, by using synthetic data, that Total Variation (TV) regularization significantly reduces leakage of the vertical velocity in the epsilon model during the inversion. Finally, we show an application on field data from the Gulf of Mexico where the flatness of the common image gathers is significantly improved.

Introduction

In most geologic scenarios accounting for anisotropy is crucial for successful application of FWI. Although using FWI to estimate the vertical velocity is a routine practice, jointly updating velocity and anisotropic parameters by inversion has proven to be difficult. The challenge arises from the coupled effect that vertical velocity and anisotropy produce on the surface seismic response. This problem is referred to as leakage or crosstalk between parameters and is shared by inversion methods that rely only on surface seismic data (either ray tomography or FWI).

In multi-parameter FWI for acoustic anisotropic media, a variety of parameterizations and inversion strategies have been proposed (e.g., Plessix and Cao, 2011; Alkhalifah and Plessix, 2014; Cheng et al., 2015). In this context, a useful conclusion obtained by some authors (e.g., Plessix and Cao, 2011, Debens et al., 2015) is that in order to match the kinematics of the propagating waves at large offsets, it is enough to produce a low-resolution ε field. In addition, field data applications call for different data selection strategies that could be used to reduce leakage of the different medium properties. Cheng et al. (2016) concluded that the diving waves sense in a similar way both vertical velocity v_z and ε. They recommend the inclusion of reflected events in the FWI when v_z is inverted to provide independent information and reduce leakage in the two-parameter updates. Another approach is to filter the gradient by wavenumber content (Alkhalifah, 2015); this also has the potential to reduce leakage of the different model parameters.

Here, we use a parameterization consisting of vertical velocity v_z and Thomsen parameters ε and δ to describe an acoustic TI medium (VTI or TTI). We assume that the δ field is obtained from other types of data (e.g., well logs, VSP) and we invert for v_z and ε. The resulting v_z sensitivity kernel for this parameterization is similar in form to one derived for inverting isotropic acoustic velocity (Ramos-Martinez et al., 2016). This velocity kernel targets long-wavelength model updates (diving waves and "rabbit ears") because it suppresses the specular reflectivity in the gradient (migration isochrones). To compute the gradient for ε, we derive a new adjoint-state equation within the framework of our pseudo-analytical extrapolator. The result is a cross correlation-type gradient that enables updating all wavelength components of the ε field. To enhance the long-wavelength features during model updating, we apply TV regularization to the inversion. Our TV implementation is based on the split Bregman method that provides an effective algorithm for solving L^1 optimization problems. TV regularization enables the incorporation of a priori information on the smoothness of the ε parameter. Thus, it reduces leakage of v_z into the ε update. From a synthetic example, we validate the ε kernel and the role of the regularization in the minimization of leakage between parameters. We also show a successful application in a field dual-sensor dataset from the Gulf of Mexico, performing an alternating inversion between v_z and ε.

Theory

FWI solves a nonlinear inverse problem by iteratively updating the model parameters to improve the match between modeled and recorded field data. This is often accomplished by minimizing an objective function typically in a least-squares sense. The inversion algorithm computes the model updates as a scaled representation of the objective function gradient with respect to each model parameter. This gradient depends on the sensitivity kernel of the different parameters describing the earth model, either isotropic or anisotropic.

Our FWI solves the acoustic wave equation by using the pseudo-analytical method introduced by Etgen and Brandsberg-Dahl (2009). We depart from the VTI dispersion relation:

$$\omega^2 = v_z^2 k_z^2 + v_h^2\left(k_x^2 + k_y^2\right) + \\ + \left(v_{nmo}^2 - v_h^2\right)\left(k_x^2 + k_y^2\right)k_z^2 / \left(k_x^2 + k_y^2 + k_z^2\right) \tag{1}$$

where ω is the angular frequency, v_z, v_h and v_{nmo} are the vertical, horizontal and NMO velocities, k_i are the wavenumber vector components along the space coordinates $\mathbf{x}$. After transforming equation (1) to space-time domain, and solving the time derivative using a second-order finite-difference approximation, we obtain the following time-stepping scheme:

$$\begin{aligned} S(\mathbf{x},t+\Delta t) &= 2S(\mathbf{x},t) - S(\mathbf{x},t-\Delta t) + \\ &+ \Delta t^2 v_z^2 FT^{-1}\left[f_z(k_z)S(\mathbf{k},t)\right] + \\ &+ \Delta t^2 v_h^2 FT^{-1}\left[f_h(k_x,k_y)S(\mathbf{k},t)\right] + \\ &+ \Delta t^2\left(v_{nmo}^2(\mathbf{x}) - v_h^2(\mathbf{x})\right)FT^{-1}\left[f_n(k_x,k_y,k_z)S(\mathbf{k},t)\right] + s_f \end{aligned} \quad (2)$$

where S is the forward-propagated wavefield, the point-source term with source wavelet $w(t)$ is $s_f = \delta(\mathbf{x}-\mathbf{x}_s)w(t)$, and FT^{-1} stands for inverse Fourier Transform. The differential operators f_i are combinations of the normalized pseudo-Laplacian operator (Chiu and Stoffa, 2011), which corrects for the inaccuracy produced by of the second-order finite-difference approximation to the time derivative with time step Δt. For example:

$$f_z(k_z) = \frac{2\cos(v_v|\mathbf{k}|\Delta t) - 2}{\Delta t^2 v_z^2 |\mathbf{k}|^2}\left(k_z^2\right) \quad (3)$$

$$f_h(k_x,k_y) = \frac{2\cos(v_v|\mathbf{k}|\Delta t) - 2}{\Delta t^2 v_z^2 |\mathbf{k}|^2}\left(k_y^2 + k_y^2\right) \quad (4)$$

The adjoint-state equation corresponding to the state equation (2) has the form

$$\begin{aligned} R(\mathbf{x},t+\Delta t) &= 2R(\mathbf{x},t) - R(\mathbf{x},t-\Delta t) + \\ &+ \Delta t\ v_z^2 FT^{-1}\left[f_z(k_z)R(\mathbf{k},t)\right] + \\ &+ \Delta t^2 v_h^2 FT^{-1}\left[f_h(k_x,k_y)FT\{(1+2\varepsilon)R(\mathbf{x},t)\}\right] + \\ &+ \Delta t^2\left(v_{nmo}^2(\mathbf{x}) - v_h^2(\mathbf{x})\right)FT^{-1}\left[f_n(k_x,k_y,k_z)FT\{2(\varepsilon-\delta)R(\mathbf{x},t)\}\right] + \\ &+ \bar{s}_f \end{aligned} \quad (5)$$

where R is the back-propagated wavefield, ε and δ are the Thomsen parameters. The adjoint source term is the residual wavefield $\bar{s} = S(\mathbf{x}_R,t) - d(\mathbf{x}_R,t)$.

For a parameterization consisting of v_z, acoustic impedance (computed from v_z), ε and δ, and assuming that δ is constant during the inversion, the gradients for v_z and ε have the following form

$$G_{v_z}(\mathbf{x}) = \frac{1}{2A_{v_z}(\mathbf{x})}\left[\int_t \left[\begin{aligned} &-W_1(\mathbf{x},t)\nabla S(\mathbf{x},t)\cdot\nabla R(\mathbf{x},T-t) \\ &+W_2(\mathbf{x},t)\frac{1}{v_z^2(\mathbf{x})}\frac{\partial S(\mathbf{x},t)}{\partial t}\cdot\frac{\partial R(\mathbf{x},T-t)}{\partial t} \end{aligned}\right]dt\right] \quad (6)$$

$$G_\varepsilon(\mathbf{x}) = -\frac{2}{A_\varepsilon(\mathbf{x})}\int\left[FT^{-1}\left\{\begin{aligned} &f_h(k_x,k_y)S(\mathbf{k},t) \\ &-(f_n(k_x,k_y,k_z)S(\mathbf{k},t) \end{aligned}\right\}\right]\cdot R(\mathbf{x},T-t)dt \quad (7)$$

where A_{v_z} and A_ε are source illumination terms, and W_i are dynamic weights designed to optimally suppress the high wavenumber components in the gradient and thus produce long-wavelength updates (Ramos-Martínez et al., 2016).

On the other hand, the epsilon gradient (equation 7) is built by crosscorrelating a modified version of the source forward propagated wavefield solved from equation 2, and the back-propagated residual wavefield solved from the adjoint-state equation 5. To enhance long-wavelength updates with this gradient, we apply TV regularization (Guo and de Hoop, 2013). In our TV implementation, we use split Bregman iterations (Goldstein and Osher, 2009), an effective algorithm for solving L^1 optimization problems. The result is a computationally efficient and accurate implementation

For simplicity, here we show the forms of the adjoint-state equation and the epsilon gradient for a VTI medium. However, these equations can be easily extended for the TTI case by a rotation of the wavenumbers to match the tilted symmetry axes.

Examples

We use a synthetic example to illustrate the performance of the new $\boldsymbol{\varepsilon}$ gradient and the role of TV regularization in reducing leakage between the inverted parameters. Figure 1a shows the $\boldsymbol{v_z}$ model, which is a modified version of the SEAM sediment model, including a low velocity anomaly in the centre. Figure 1b shows the difference between the true and the starting epsilon fields. The inversion for $\boldsymbol{\varepsilon}$ assumes that the velocity field is exact. Figure 1c shows the difference between the inverted and the started $\boldsymbol{\varepsilon}$ models without regularization. Even for the case of exact vertical velocity, leakage of the velocity anomaly at the centre is observed in the $\boldsymbol{\varepsilon}$ updates. Figure 1d shows the $\boldsymbol{\varepsilon}$ difference for the inversion using TV regularization. The velocity anomaly leakage is significantly reduced and the updates resemble the true perturbations shown in Figure 1b.

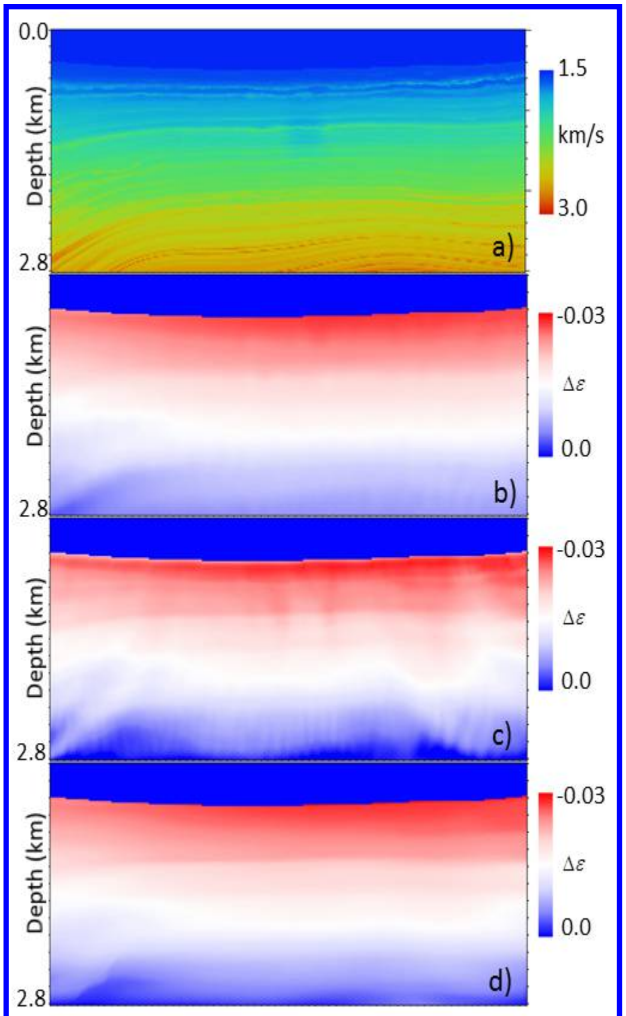

Figure 1. a) Vertical velocity and b) difference between the true and starting epsilon models for the synthetic example. Difference between the inverted and starting epsilon models after inversion without (c) and with (d) TV regularization.

We tested the multi-parameter FWI on a dual-sensor field data set from the De Soto Canyon area in the Gulf of Mexico. Figure 2 shows sample shot records used in the inversion. The data has a maximum offset of 12 km. The starting vertical velocity model is shown in Figure 3a. The initial ε and δ models are zero in the water column and constant in the sediments with values of 0.08 and 0.04 respectively. We use a maximum frequency of 7 Hz performing an alternating inversion between v_z and ε. Figure 3b and Figure 3c show the final v_z and ε models. Figures 4a and 4b illustrate clear improvement in the flatness of the Kirchhoff offset gathers after inversion.

Conclusions

We introduce a practical FWI approach to retrieve the long wavelength updates in an acoustic anisotropic medium with transverse isotropy. We use a parameterization consisting of vertical velocity, epsilon and delta, to obtain the sensitivity kernels for vertical velocity and epsilon. First, we update the vertical velocity from a long-wavelength gradient which is similar in form to one derived from the isotropic velocity sensitivity kernel that is able to suppress migration isochrones. Then, long-wavelength epsilon updates are obtained by TV regularization, which significantly reduces leakage of the velocity imprint in the epsilon field. We show a successful application of the approach to a dual-sensor dataset acquired in the Gulf of Mexico. Results are validated by the improvement of the flatness of the image gathers using the inverted models after two stages of cascaded inversions of vertical inversion and epsilon.

Acknowledments

We thank PGS for permission to publish the results and PGS MultiClient for providing the data. We thank Nizar Chemingui, Sean Crawley, Jan Kirkebo, Dan Whitmore Faqi Liu and Volker Dirks for useful suggestions

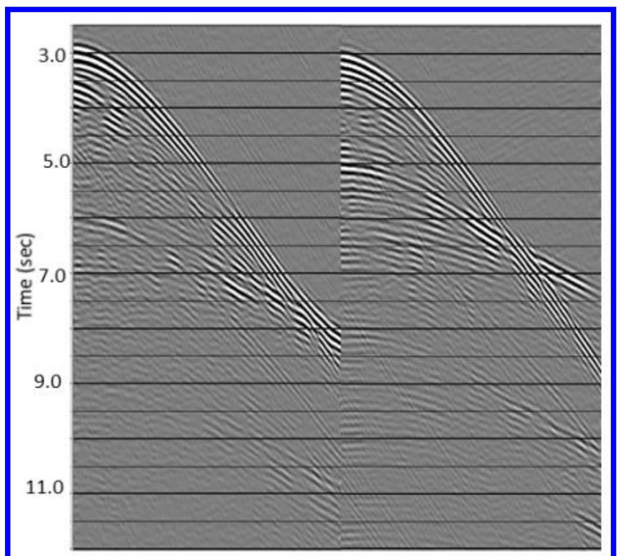

Figure 2. Sample shot records corresponding to a dataset acquired with dual-sensor technology in the De Soto Canyon area, Gulf of Mexico. Maximum offset is 12 km.

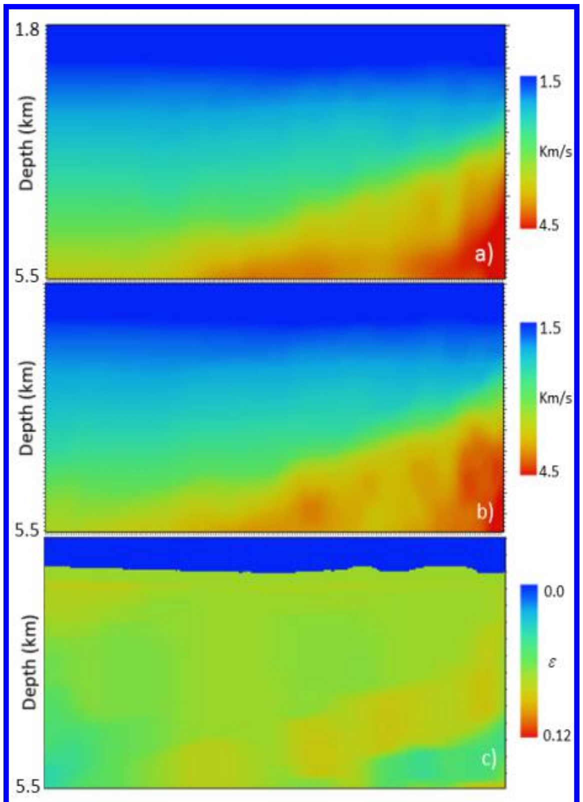

Figure 3. a) Starting vertical velocity model for the Gulf of Mexico field data example; epsilon and delta models are zero in the water column, and homogeneous from the water bottom with values 0.08 and 0.04, respectively. b) Vertical velocity and c) epsilon models after cascade inversion.

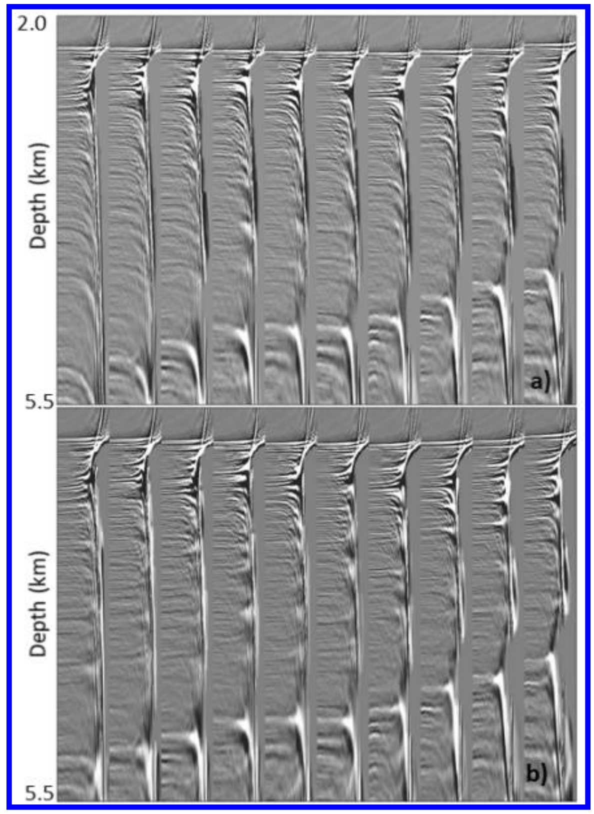

Figure 4. Kirchhoff offset gathers for the a) starting and b) inverted vertical velocity and epsilon models corresponding to the field data example of Gulf of Mexico.

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